Simulation of Pedestrian Crowds in Normal and Evacuation Situations

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Starting with a short review of the available literature in the field of pedestrian and evacuation research, an overview is given over the observed collective phenomena in pedestrian crowds. This includes lane formation in corridors and oscillations at bottlenecks in normal situations, while different kinds of blocked states are produced in panic situations. By means of molecular-dynamic-like microsimulations based on a generalized force model of interactive pedestrian dynamics, the spatio-temporal patterns in pedestrian crowds are successfully reproduced and interpreted as self-organized phenomena. In contrast to previous socio-psychological approaches, this allows a physical understanding of the observations. Despite the significantly different phenomena occuring in normal and panic situations, the main effects can be described by a unified model containing only well interpretable and plausible terms. The transition between the "rational" normal behavior and the apparently "irrational" panic behavior is controlled by a single parameter, the "nervousness", which influences fluctuation strengths, desired speeds, and the tendency of herding. Thereby, it causes paradoxial effects like "freezing by heating", "faster is slower", and the ignorance of available exits. Nevertheless, there are measures to improve pedestrian flows, both in normal and panic situations. For example, the suitable placement of columns can help, although they reduce the accessible space.

1 Introduction

The various collective phenomena observed in pedestrian crowds have recently attracted the interest of a rapidly increasing number of scientists. In this review, we will always distinguish the dynamics of pedestrians in normal and panic situations. Since both problems are characterized by different characteristic phenomena, they have often been investigated by different scientific communities. However, as we will show in the following, they can be treated in a consistent way by one and the same pedestrian model.

1.1 Research in normal pedestrian behavior

Pedestrian crowds have been empirically studied for more than four decades now [1–8]. The evaluation methods applied were based on direct observation, photographs, and time-lapse films. Apart from behavioral investigations [9,10], the main goal of these studies was to develop a *level-of-service concept* [11–13], *design elements* of pedestrian facilities [14–19], or *planning guidelines* [20–24]. The latter have usually the form of *regression relations*, which are, however, not very well suited for the prediction of pedestrian flows in pedestrian zones and buildings with an exceptional architecture, or in extreme conditions such as evacuation. Therefore, a number of simulation models have been proposed, e.g. *queueing models* [25–27], *transition matrix models* [28], and *stochastic models* [29,30], which are partly related to each other. In addition, there are models for the *route choice behavior* of pedestrians [31–34].

None of these concepts adequately takes into account the self-organization effects occuring in pedestrian crowds. These may, however, lead to unexpected obstructions due to mutual disturbances of pedestrian flows. More promising with regard to this is the approach by Henderson. He conjectured that pedestrian crowds behave similar to gases or fluids ([35–38], see also [39,40]). This could be partly confirmed (see Sec. 2.3). However, a realistic gas-kinetic or fluid-dynamic theory for pedestrians must contain corrections due to their particular interactions (i.e. avoidance and deceleration maneuvers) which, of course, do not obey momentum and energy conservation. Although such a theory can be actually formulated [34,41,42], for practical applications a direct simulation of *individual* pedestrian motion is favourable, since this is more flexible. As a consequence, current research focusses on the *microsimulation* of pedestrian crowds, which also allows us to consider incoordination by *excluded volume effects* related to the discrete, "granular" structure of pedestrian flows. In this connection, a *behavioral force model* of individual pedestrian dynamics has been developed [18,43–53] (see Sec. 3). A discrete and simple forerunner of this model was proposed by Gipps and Marksjö (1985). We also like to mention recent *cellular automata* of pedestrian dynamics [54–63], and *AI-based models* [64–67].

1.2 Evacuation and panic research

Computer models for emergency and evacuation situations have been developed as well [62,68–76]. Most research into panics, however, has been of empirical nature (see, e.g. Refs. [77–80]), carried out by social psychologists and others.

With some exceptions, panics are observed in cases of scarce or dwindling resources [81,77], which are either required for survival or anxiously desired. They are usually distinguished into escape panics ("stampedes", bank or stock market panics) and acquisitive panics ("crazes", speculative manias) [82,83], but in some cases this classification is questionable [84].

It is often stated that panicking people are obsessed by short-term personal interests uncontrolled by social and cultural constraints [77,82]. This is possibly a result of the reduced attention in situations of fear [77], which also causes that options like side exits are mostly ignored [78]. It is, however, frequently attributed to social contagion [77,79,81–89], i.e., a transition from individual to mass psychology, in which individuals transfer control over their actions to others [83], leading to conformity [90]. This "herding behavior" is in some sense irrational, as it often leads to bad overall results like dangerous overcrowding and slower escape [83,84,78]. In this way, herding behavior can increase the fatalities or, more generally, the damage in the crisis faced.

The various socio-psychological theories for this contagion assume hypnotic effects, rapport, mutual excitation of a primordial instinct, circular reactions, social facilitation (see the summary by Brown [88]), or the emergence of normative support for selfish behavior [89]. Brown [88] and Coleman [83] add another explanation related to the prisoner's dilemma [91,92] or common goods dilemma [93], showing that it is reasonable to make one's subsequent actions contingent upon those of others, but the socially favourable behavior of walking orderly is unstable, which normally gives rise to rushing by everyone. These thoughtful considerations are well compatible with many aspects discussed above and with the classical experiments by Mintz [81], which showed that jamming in escape situations depends on the reward structure ("payoff matrix").

Nevertheless and despite of the frequent reports in the media and many published investigations of crowd disasters (see Table 1), a quantitative understanding of the observed phenomena in panic stampedes has been lacking. In this study, we will add another aspect to the explanation of panics by simulating a computer model for the crowd dynamics of pedestrians.

2 Observations

2.1 Normal situations

We have investigated pedestrian motion for several years and evaluated a number of video films. Despite the sometimes more or less "chaotic" appearance of individual pedestrian behavior, one can find regularities, some of which become best visible in time-lapse films like the ones produced by Arns [94]. While describing these, we also summarize results of other pedestrian studies and observations [18,19,45,95]:

1. Pedestrians feel a strong aversion to taking detours or moving opposite to the desired walking direction, even if the direct way is crowded. However, there is also some evidence that pedestrians normally choose the *fastest* route to their next destination, but not the *shortest one* [96]. In general, pedestrians take into account detours as well as the comfort of walking, thereby minimizing the effort to reach their destination [97]. Their ways can be approximated by polygons.
2. Pedestrians prefer to walk with an individual desired speed, which corresponds to the most comfortable (i.e. least energy-consuming) walking speed (see Ref. [8]) as long as it is not necessary to go faster in order to reach the destination in time. The desired speeds within pedestrian crowds are Gaussian distributed with a mean value of approximately 1.34 m/s and a standard deviation of about 0.26 m/s [35]. However, the average speed depends on the situation [21], sex and age, the time of the day, the purpose of the trip, the surrounding, etc. [8].
3. Pedestrians keep a certain distance to other pedestrians and borders (of streets, walls, and obstacles; see [22,24]). This distance is smaller the more a pedestrian is in a hurry, and it decreases with growing pedestrian density.

Table 1: Incomplete list of major crowd disasters after J. F. Dickie in Ref. [98], http://ourworld.compuserve.com/homepages/G_Keith_Still/disaster.htm, http://SportsIllustrated.CNN.com/soccer/world/news/2000/07/09/stadium_disasters_ap/, and other internet sources (from [99]). The number of injured people was usually a multiple of the fatalities.

Date	Place	Venue	Deaths	Injured	Reason
1863	Santiago, Chile	Church	2000		
1881	Vienna, Austria	Theatre	570		
1883	Sunderland, UK	Theatre	182		
1902	Ibrox, UK	Stadium	26	517	Collapse of West Stand
1903	Chicago, USA	Theatre	602		
1943	London, UK	Subway Station	173		Stampede while air raid
1946	Bolton, UK	Stadium	33	400	Collapse of a wall
1955	Santiago, Chile	Stadium	6		Fans trying to force their way into the stadium
1961	Rio de Janeiro, Brazil	Circus	250		
1964	Lima, Peru	Stadium	318	500	Goal disallowed
1967	Kayseri, Turkey	Stadium	40		
1968	Buenos Aires, Argentina	Stadium	75	150	Fans fleeing from fire
1970	St. Laurent-du-Pont, France	Dance Hall	146		
1971	Ibrox, UK	Stadium	66	140	Collapse of barriers
1971	Salvador, Brazil	Stadium	4	1500	Fight and wild rush
1974	Cairo, Egypt	Stadium	48		Crowds break barriers
1976	Port-au-Prince, Haiti	Stadium	2		Firecracker
1979	Nigeria	Stadium	24	27	Light failure
1979	Cincinatti, USA	Stadium	11		Fans trying to force their way into the stadium
1981	Piraeus, Greece	Stadium	24		Rush of leaving fans
1981	Sheffield, UK	Stadium		38	Crowd surge
1982	Cali, Columbia	Stadium	24	250	Provocation by drunken fans
1982	Moscow, USSR	Stadium	340		Re-entering fans after last minute goal
1985	Bradford, UK	Stadium	56		Fire in wooden terrace section
1985	Mexico City, Mexico	Stadium	10	29	Fans trying to force their way into the stadium
1985	Brussels, Belgium	Stadium	38	> 400	Riots break out
1987	Tripoli, Libya	Stadium	2	16	Collapse of a wall
1988	Katmandu, Nepal	Stadium	93	> 100	Stampede due to hailstorm
1989	Hillsborough, Sheffield, UK	Stadium	96		Fans trying to force their way into the stadium
1990	Mecca, Saudi Arabia	Pedestrian Tunnel	1425		Overcrowding
1991	Orkney, South Africa	Stadium	> 40		Fans trying to escape fighting

Date	Place	Venue	Deaths	Injured	Reason
1991	New York, USA	Stadium	9		Overcrowding at concert
1992	Rio de Janeiro, Brazil	Stadium		50	Part of the fence giving way
1992	Bastia, Corsica	Stadium	17	1900	
1994	Mecca, Saudi Arabia		270		"Stoning the devil" ritual
1995	New Delhi, India	Tent	≈ 400	> 100	Indian fire
1996	Lusaka, Zambia	Stadium	9	78	Stampede after Zambia's victory over Sudan
1996	Tembisa, South Africa	Railway Station	15	> 20	Electric cattle prods used by security guards
1996	Guatemala City, Guatemala	Stadium	80	180	Fans trying to force their way into the stadium
1997	Las Vegas, USA	Hotel	1	50	Gunshot
1997	Düsseldorf, Germany	Stadium	1	> 300	Overcrowding at concert
1998	Dhaka, Bangladesh	Multi-Storey Building	1	15	Fire stampede
1998	Mecca, Saudi Arabia		107		"Stoning the devil" ritual
1998	Harare, Zimbabwe	Stadium	4	10	Spectators scrambled for seats
1998	Manila, Phillipines	Presidential Action Center	2		Large crowd waiting for jobs and housing
1998	Chervonohrad, Ukraine	Cinema	4		Stampede due to in- and outcoming children
1998	Lima, Peru	Disco	9	7	Tear gas
1999	Minsk, Belarus	Subway Station	51	150	Heavy rain at rock concert
1999	Kerala, India	Hindu Shrine	> 50		Collapse of parts of the shrine
1999	Benin, Nigeria	Religious Place	14		Stampede at a Christian revivalist rally
1999	Innsbruck, Austria	Stadium	5	25	Fans re-entering the stadium?
2000	Kaloroa, Bangladesh	Examination Place	5		Stampede to enter an examination hall
2000	Mecca, Saudi Arabia	Holy Place	2	4	Pilgrim overcrowding
2000	Durban, South Africa	Disco	13	44	Tear gas
2000	Chiaquelane, Mozambique	Chiaquelane Camp	5	10	Aid chaos
2000	Lisbon, Portugal	Nightclub	7	65	Poisonous gas bombs
2000	Seville, Spain			30	Good Friday procession
2000	Monrovia, Liberia	Stadium	3		Fans trying to force their way into the stadium
2000	Lahore, Pakistan	Circus	8	3	Guards used batons
2000	Addis Abeba, Ethiopia	Memorial Place	14		Children trying to cover from a rainstorm
2000	Roskilde, Denmark	Stadium	8	25	Failure of loud speakers
2000	Harare, Zimbabwe	Stadium	12		Tear gas
2000	São Januário, Brazil	Stadium		200	Oversold stadium

Resting individuals (waiting on a railway platform for a train, sitting in a dining hall, or lying at a beach) are uniformly distributed over the available area if there are no acquaintances among the individuals. Pedestrian density increases (i.e. interpersonal distances lessen) around particularly attractive places. It decreases with growing velocity variance, e.g., on a dance floor [41,18]. Individuals knowing each other may form groups which are entities behaving similar to single pedestrians. Group sizes are Poisson distributed [100–102].

2.2 Panic situations

Panic stampede is one of the most tragic collective behaviors [79,81–83,85–89], as it often leads to the death of people who are either crushed or trampled down by others. While this behavior is comprehensible in life-threatening situations like fires in crowded buildings [77,78], it is hard to understand in cases of a rush for good seats at a pop concert [84] or without any obvious reasons. Unfortunately, the frequency of such disasters is increasing [84] (see Table 1), as growing population densities combined with easier transportation lead to greater mass events like pop concerts, sporting events, and demonstrations. Nevertheless, systematic studies of panics [81,103] are rare [77,82,84], and there is a scarcity of quantitative theories capable of predicting the dynamics of human crowds [62,68,69,72,73,76]. In spite of this, the following features appear to be typical [52,53]:

1. In situations of escape panics, individuals are getting nervous, i.e. they tend to develop blind actionism.
2. People try to move considerably faster than normal [21].
3. Individuals start pushing, and interactions among people become physical in nature.
4. Moving and, in particular, passing of a bottleneck frequently becomes incoordinated [81].
5. At exits, jams are building up [81]. Sometimes, arching and clogging are observed [21], see Fig. 1.
6. The physical interactions in jammed crowds add up and can cause dangerous pressures up to 4,500 Newtons per meter [78,98], which can bend steel barriers or tear down brick walls.
7. Escape is slowed down by fallen or injured people turning into "obstacles".
8. People tend to show herding behavior, i.e., to do what other people do [77,86].
9. Alternative exits are often overlooked or not efficiently used in escape situations [77,78].

The following quotations give a more personal impression of the conditions during escape panics:

1. "They just kept pushin' forward and they would just walk right on top of you, just trample over ya like you were a piece of the ground." (After the panic at "The Who Concert Stampede" in Cincinatti.)
2. "People were climbin' over people ta get in ... an' at one point I almost started hittin' 'em, because I could not believe the animal, animalistic ways of the people, you know, nobody cared." (After the panic at "The Who Concert Stampede".)
3. "Smaller people began passing out. I attempted to lift one girl up and above to be passed back ... After several tries I was unsuccessful and near exhaustion." (After the panic at "The Who Concert Stampede".)

Figure 1: Panicking football fans trying to escape the football stadium in Sheffield. Because of a clogging effect, it is difficult to pass the open door.

4. I "couldn't see the floor because of the thickness of the smoke." (After the "Hilton Hotel Fire" in Las Vegas.)
5. "The club had two exits, but the young people had access to only one, said Narend Singh, provincial minister for agriculture and environmental affairs. However, the club's owner, Rajan Naidoo, said the club had four exits, and that all were open. 'I think the children panicked and headed for the main entrance where they initially came in,' he said." (After the "Durban Disco Stampede".)

2.3 Analogies with gases, fluids, and granular media

When the density is low, pedestrians can move freely, and crowd dynamics can be compared with the behavior of gases. At medium and high densities, the motion of pedestrian crowds shows some striking analogies with the motion of fluids and granular flows:

1. Footprints of pedestrians in snow look similar to streamlines of fluids [34].
2. At borderlines between opposite directions of walking one can observe "viscous fingering" [104,105].
3. The emergence of pedestrian streams through standing crowds [94,18,19,45] appears analogous to the formation of river beds [106–108] (see Fig. 2).
4. Similar to segregation or stratification phenomena in granular media [109,110], pedestrians spontaneously organize in lanes of uniform walking direction, if the pedestrian density is high enough [2,18,19,43,45] (see Fig. 3).
5. At bottlenecks (e.g. corridors, staircases, or doors), the passing direction of pedestrians oscillates [46,47]. This may be compared to the "saline oscillator" [111] or the granular "ticking hour glass" [112,113].

Figure 2: The long-term photograph of a standing crowd in front of a cinema taken by Thomas Arns shows that crossing pedestrians form a river-like stream (from [18,19,45]).

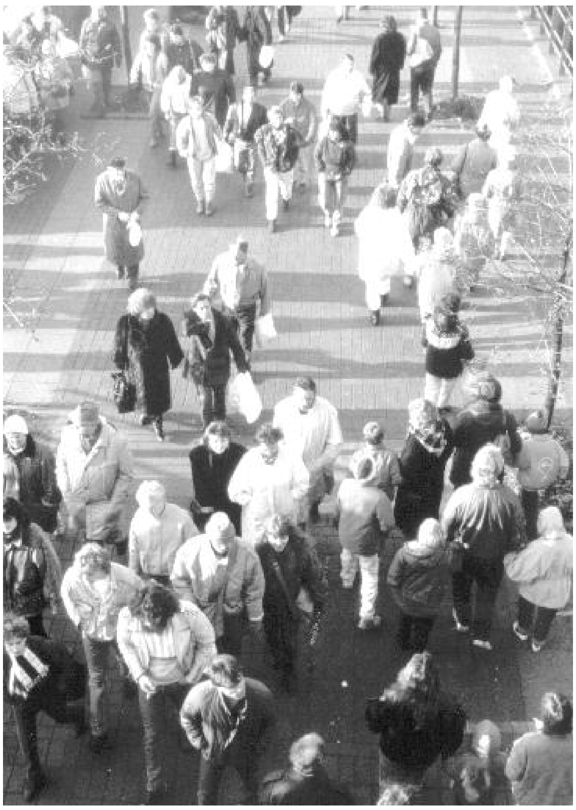

Figure 3: At sufficiently high densities, pedestrians form lanes of uniform walking direction (from [18,19,45]).

6. One can find the propagation of shock waves in dense pedestrian crowds pushing forward (see also [114]).
7. The arching and clogging in panicking crowds [53] is similar to the outflow of rough granular media through small openings [115,116].

In summary, one could say that fluid-dynamic analogies work well in normal situations, while granular aspects become important in panic situations.

3 Generalized force model of pedestrian dynamics

3.1 The social force concept

Human behavior often seems to be "chaotic", irregular, and unpredictable. So, why and under what conditions can we model it by means of forces? First of all, we need to be confronted with a phenomenon of motion in some (quasi-)continuous space, which may be also an abstract behavioral space or opinion scale [34,117–119]. Moreover, it is favourable to have a system where the fluctuations due to unknown influences are not large compared to the systematic, deterministic part of motion. This is usually the case in pedestrian and vehicle traffic, where people are confronted with standard situations and react automatically rather than taking complicated decisions between various possible alternatives. For example, an experienced driver would not have to think about the detailled actions to be taken when turning, accelerating, or changing lanes.

This automatic behavior can be interpreted as the result of a *learning process* based on trial and error [19], which can be simulated with *evolutionary algorithms* [120–123]. For example, pedestrians have a preferred side of walking [2,4,8], since an asymmetrical avoidance behavior turns out to be profitable [54]. The related *formation of a behavioral convention* can be described by means of *evolutionary game theory* [34,43,119,124–127].

Another requirement is the vectorial additivity of the separate force terms reflecting different environmental influences. This is probably an approximation, but there is some experimental evidence for it. Based on quantitative measurements for animals and test persons subject to separately or simultaneously applied stimuli of different nature and strength, one could show that the behavior in conflict situations can be described by a superposition of forces [128–130]. This fits well into a concept by Lewin [131], according to which behavioral changes are guided by so-called *social fields* or *social forces*, which has been put into mathematical terms by Helbing [34,43,47,117–119]. In some cases, social or *behavioral forces*, which determine the amount and direction of systematic behavioral changes, can be expressed as gradients of dynamically varying potentials, which reflect the social or behavioral fields resulting from the interactions of individuals. The behavioral force concept was applied to opinion formation [34,117–119] and migration [34,118,119], but it was particularly successful in the description of pedestrian and vehicle traffic [43,46,47,132,133].

For reliable simulations of pedestrian crowds we do not need to know whether a certain pedestrian, say, turns to the right at the next intersection. It is sufficient to have a good estimate what percentage of pedestrians turns to the right. This can be either empirically measured or calculated by means of route choice models like the one by Borgers and Timmermans [31,32]. In some sense, the uncertainty about the individual behaviors is averaged out at the macroscopic level of description, as in fluid dynamics. Nevertheless, instead of a fluid-dynamic model, we will use the more flexible microscopic simulation approach based on the generalized force concept. According to this, the temporal change of the location $\boldsymbol{x}_i(t)$ of pedestrian i obeys the equation of motion

$$\frac{d\boldsymbol{x}_i(t)}{dt} = \boldsymbol{v}_i(t) \, . \tag{1}$$

Moreover, if $\boldsymbol{f}_i(t)$ denotes the sum of forces influencing pedestrian i, m_i is the mass of pedestrian i, and $\boldsymbol{\xi}_i(t)$ are individual fluctuations reflecting unsystematic behavioral variations, the velocity changes are given by the *acceleration equation*

$$m_i \frac{d\boldsymbol{v}_i}{dt} = \boldsymbol{f}_i(t) + \boldsymbol{\xi}_i(t) \, . \tag{2}$$

Particular advantages of this approach are that we can take into account

- the flexible usage of space by pedestrians, requiring a (quasi-)continous treatment of motion, and
- excluded volume effects due to granular properties of panicking pedestrian crowds.

It turns out that these points are essential to reproduce the above mentioned phenomena in a natural way.

3.2 Social force model for normal pedestrian dynamics

We will now describe the different motivations of and influences on a pedestrian i by separate force terms. First of all, the desire to adapt the actual velocity $\boldsymbol{v}_i(t)$ to the desired speed v_i^0 and direction $\boldsymbol{e}_i^0(t)$ within a certain "relaxation time" τ_i is reflected by the *acceleration term* $[v_i^0(t)\boldsymbol{e}_i^0(t) - \boldsymbol{v}_i(t)]/\tau_i$. Herein, the contribution $v_i^0(t)\boldsymbol{e}_i^0(t)/\tau_i$ can be interpreted as *driving term*, while $-\boldsymbol{v}_i(t)/\tau_i$ has the meaning of a *friction term* with friction coefficient $1/\tau_i$.

Next, the tendency of pedestrians to keep a certain distance to other pedestrians ("territorial effect") may be described by repulsive social forces

$$\boldsymbol{f}_{ij}^{\text{soc}}(t) = A_i \exp[(r_{ij} - d_{ij})/B_i] \boldsymbol{n}_{ij} \left(\lambda_i + (1-\lambda_i) \frac{1+\cos(\varphi_{ij})}{2} \right) \, . \tag{3}$$

Herein, A_i denotes the interaction strength and B_i the range of the repulsive interactions, which are culture-dependent and individual parameters. $d_{ij}(t) = \|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)\|$ is the distance between the centers of mass of pedestrians i and j, $r_{ij} = (r_i + r_j)$ the sum of their radii r_i and r_j, and

$$\boldsymbol{n}_{ij}(t) = \big(n_{ij}^1(t), n_{ij}^2(t) \big) = \frac{\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)}{d_{ij}(t)} \tag{4}$$

the normalized vector pointing from pedestrian j to i. Finally, with the choice $\lambda_i < 1$, we can reflect the anisotropic character of pedestrian interaction. In other words, with the parameter λ_i we can model that the situation in front of a pedestrian has a larger impact on his or her behavior than things happening behind. The angle $\varphi_{ij}(t)$ denotes the angle between the direction $\boldsymbol{e}_i(t) = \boldsymbol{v}_i(t)/\|\boldsymbol{v}_i(t)\|$ of motion and the direction $-\boldsymbol{n}_{ij}(t)$ of the object exerting the repulsive force, i.e. $\cos\varphi_{ij}(t) = -\boldsymbol{n}_{ij}(t)\cdot\boldsymbol{e}_i(t)$. One may, of course, take into account other details such as a velocity-dependence of the forces and non-circular shaped pedestrian bodies, but this does not have *qualitative* effects on the dynamical phenomena resulting in the simulations. In fact, most observed self-organization phenomena are quite insensitive to the specification of the interaction forces, as different studies have shown [47,49,51–53].

In addition, we may also take into account time-dependent attractive interactions towards window displays, sights, or special attractions k by social forces $\boldsymbol{f}_{ik}^{\text{att}}(t)$ of the type (3). However, compared with repulsive interactions, the corresponding interaction range B_{ik} is usually larger and the strength parameter $A_{ik}(t)$ typically smaller, negative, and time-dependent. Additionally, the joining behavior [134] of families, friends, or tourist groups can be reflected by forces of the type $\boldsymbol{f}_{ij}^{\text{att}}(t) = -C_{ij}\boldsymbol{n}_{ij}(t)$, which guarantee that acquainted individuals join again, after they have accidentally been separated by other pedestrians.

In summary, the force model of pedestrian motion in normal situations corresponds to Eqs. (1) and (2) with

$$\boldsymbol{f}_i(t) = \frac{v_i^0(t)\boldsymbol{e}_i^0(t) - \boldsymbol{v}_i(t)}{\tau_i} + \sum_{j(\neq i)}[\boldsymbol{f}_{ij}^{\text{soc}}(t) + \boldsymbol{f}_{ij}^{\text{att}}(t)] + \sum_b \boldsymbol{f}_{ib}(t) + \sum_k \boldsymbol{f}_{ik}^{\text{att}}(t)\,. \quad (5)$$

In the following, we will use a simplified version of this model by dropping attraction effects and assuming $\lambda_i = 0$, so that the interaction forces become isotropic and conform with Newton's 3rd law.

3.3 Force model for panicking pedestrians

Additional, physical interaction forces $\boldsymbol{f}_{ij}^{\text{ph}}$ come into play when pedestrians get so close to each other that they have physical contact ($r_{ij} \geq d_{ij}$). In this case, which is mainly relevant to panic situations, we assume also a *"body force"* $k(r_{ij} - d_{ij})\,\boldsymbol{n}_{ij}$ counteracting body compression and a *"sliding friction force"* $\kappa(r_{ij} - d_{ij})\,\Delta v_{ji}^t\,\boldsymbol{t}_{ij}$ impeding *relative* tangential motion. Inspired by the formulas for granular interactions [115,116], we assume

$$\boldsymbol{f}_{ij}^{\text{ph}}(t) = k\Theta(r_{ij} - d_{ij})\boldsymbol{n}_{ij} + \kappa\Theta(r_{ij} - d_{ij})\Delta v_{ji}^t\,\boldsymbol{t}_{ij}\,, \quad (6)$$

where the function $\Theta(z)$ is equal to its argument z, if $z \geq 0$, otherwise 0. Moreover, $\boldsymbol{t}_{ij} = (-n_{ij}^2, n_{ij}^1)$ means the tangential direction and $\Delta v_{ji}^t = (\boldsymbol{v}_j - \boldsymbol{v}_i)\cdot\boldsymbol{t}_{ij}$ the tangential velocity difference, while k and κ represent large constants.

Strictly speaking, friction effects already set in before pedestrians touch each other, because of the psychological tendency not to pass other individuals with a high relative velocity, when the distance is small. This is, however, not important for the effects we are going to reproduce later on.

The interactions with the boundaries of walls and other obstacles are treated analogously to pedestrian interactions, i.e., if $d_{ib}(t)$ means the distance to boundary

b, $\boldsymbol{n}_{ib}(t)$ denotes the direction perpendicular to it, and $\boldsymbol{t}_{ib}(t)$ the direction tangential to it, the corresponding interaction force with the boundary reads

$$\boldsymbol{f}_{ib} = \{A_i \exp[(r_i - d_{ib})/B_i] + k\Theta(r_i - d_{ib})\}\, \boldsymbol{n}_{ib} - \kappa\Theta(r_i - d_{ib})(\boldsymbol{v}_i \cdot \boldsymbol{t}_{ib})\, \boldsymbol{t}_{ib} \,. \qquad (7)$$

Finally, fire fronts are reflected by repulsive social forces similar those describing walls, but they are much stronger. The physical interactions, however, are qualitatively different, as people reached by the fire front become injured and immobile ($v_i = 0$).

4 Simulation results

The generalized force model of pedestrian dynamics has been simulated on a computer for a large number of interacting pedestrians confronted with different situations. In spite of its simplifications, it describes a lot of observed phenomena quite realistically. Especially, it allows us to explain various self-organized spatio-temporal patterns that are not externally planned, prescribed, or organized, e.g. by traffic signs, laws, or behavioral conventions. Instead, the spatio-temporal patterns discussed below emerge due to the non-linear interactions of pedestrians even without assuming strategical considerations or communication of pedestrians. Many of these collective patterns of motion are symmetry-breaking phenomena, although the model was formulated completely symmetric with respect to the right-hand and the left-hand side [18,19,45,48].

4.1 Self-organized pedestrian dynamics in normal situations

Lane formation. Our microsimulations reproduce the empirically observed *formation of lanes* consisting of pedestrians with the same desired walking direction [18,19,44,46–51] (see Fig. 4). For open boundary conditions, these lanes are dynamically varying. Their number depends on the width of the street [18,47], on pedestrian density, and on the noise level. Interestingly, one finds a *noise-induced ordering* [51,135]: Compared to small noise amplitudes, medium ones result in a more pronounced segregation (i.e., a smaller number of lanes), while large noise amplitudes lead to a "freezing by heating" effect (see Fig. 7).

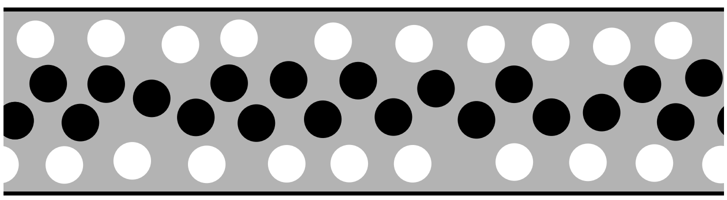

Figure 4: Formation of lanes in initially disordered pedestrian crowds with opposite walking directions (after [52,136]; cf. also [46,47,95,99,136]). White disks represent pedestrians moving from left to right, black ones move the other way round. Lane formation does not require the periodic boundary conditions applied above, see the Java applet `http://www.helbing.org/Pedestrians/Corridor.html`.

The conventional interpretation of lane formation assumes that pedestrians tend to walk on the side which is prescribed in vehicular traffic. However, the above model can explain lane formation even without assuming a preference for *any* side [51,52]. The most relevant point is the higher relative velocity of pedestrians walking in opposite directions. Pedestrians moving against the stream or in areas of mixed directions of motion will have frequent and strong interactions. In each interaction, the encountering pedestrians move a little aside in order to pass each other. This sidewards movement tends to separate oppositely moving pedestrians, which leads to segregation. The resulting collective pattern of motion minimizes the frequency and strength of avoidance maneuvers, if fluctuations are weak. Assuming identical desired velocities $v_i^0 = v_0$, the most stable configuration corresponds to a state with a minimization of the average interaction strength

$$-\frac{1}{N}\sum_{i\neq j}\tau \boldsymbol{f}_{ij}\cdot \boldsymbol{e}_i^0 \approx \frac{1}{N}\sum_i (v_0 - \boldsymbol{v}_i\cdot \boldsymbol{e}_i^0) = v_0(1-E)\,. \tag{8}$$

It is related with a maximum efficiency

$$E = \frac{1}{N}\sum_i \frac{\boldsymbol{v}_i\cdot \boldsymbol{e}_i}{v_0} \tag{9}$$

of motion corresponding to *optimal self-organization* [51], where the efficiency E with $0 \le E \le 1$ describes the average fraction of the desired speed v_0 with which pedestrians actually approach their destinations ($N = \sum_i 1$ is the respective number of pedestrians i). As a consequence, lane formation "globally" maximizes the average velocity into the respectively desired direction of motion, although the model does not even assume that pedestrians would try to optimize their behavior *locally.* This is a consequence of the symmetrical interactions among pedestrians with opposite walking directions. One can even show that a large class of driven many-particle systems, if they self-organize at all, tend to globally optimize their state [51].

Finally, note that lane formation is hard to describe by cellular automata. However, Burstedde *et al.* have recently found a way to reproduce this collective phenomenon by introducing an additional floor field inspired by trail formation models [97,137,138,45], which mimics individual intelligence. In the limit of vanishing diffusion and fast decay of the floor field, this cellular automaton is similar to Helbing and Bolay's implementation of an efficient, discretized version of the social force model [54], where the interaction effects of boundaries and pedestrians are represented by a global potential. Some interactive Java applets based on this model are available at `www.helbing.org`.

Oscillations at bottlenecks. In simulations of bottlenecks like doors, we observe oscillatory changes of the passing direction, if people do not panic [18,19,44,46–50] (see Fig. 5). Once a pedestrian is able to pass the narrowing, pedestrians with the same walking direction can easily follow. Hence, the number and "pressure" of waiting and pushing pedestrians becomes less than on the other side of the narrowing where, consequently, the chance to occupy the passage grows. This leads to a deadlock situation which is followed by a change in the passing direction.

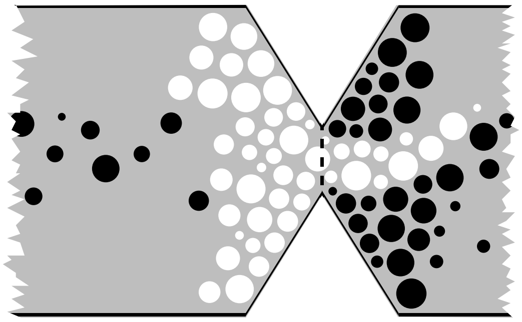

Figure 5: Oscillations of the passing direction at a bottleneck (after [95,99,136]; cf. also [46,47]). Dynamic simulations are available at `http://www.helbing.org/Pedestrians/Door.html`.

Dynamics at intersections. At intersections one is confronted with various alternating collective patterns of motion which are very short-lived and unstable. For example, phases during which the intersection is crossed in "vertical" or "horizontal" direction alternate with phases of temporary roundabout traffic (see Fig. 6) [46,44,18,49,50,48,19]. This self-organized round-about traffic is similar to the emergent rotation found for self-driven particles [139]. It is connected with small detours but decreases the frequency of necessary deceleration, stopping, and avoidance maneuvers considerably, so that pedestrian motion becomes more efficient on average.

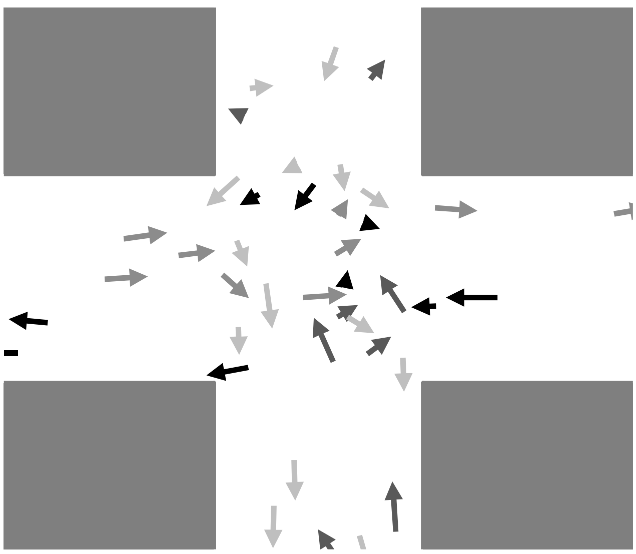

Figure 6: Self-organized, short-lived roundabout traffic in intersecting pedestrian streams (from [18,19,44,48,95]; see also [46,49,50]).

4.2 Collective phenomena in panic situations

In panic situations (e.g. in some cases of emergency evacuation) the following characteristic features of pedestrian behavior are often observed:

1. People are getting nervous, resulting in a higher level of fluctuations.
2. They are trying to escape from the source of panic, which can be reflected by a significantly higher desired velocity.
3. Individuals in complex situations, who do not know what is the right thing to do, orient at the actions of their neighbours, i.e. they tend to do what other people do. We will describe this by an additional herding interaction, but attractive interactions have probably a similar effect.

We will now discuss the fundamental collective effects which fluctuations, increased desired velocities, and herding behavior can have. In contrast to other approaches, we do not assume or imply that individuals in panic or emergency situations would behave relentless and asocial, although they sometimes do.

"Freezing by heating". The effect of getting nervous has been investigated in Ref. [52]. Let us assume the individual level of fluctuations is given by

$$\eta_i = (1 - n_i)\eta_0 + n_i\eta_{\max} \,, \tag{10}$$

where n_i with $0 \le n_i \le 1$ measures the nervousness of pedestrian i. The parameter η_0 means the normal and $\eta_{\max}$ the maximum fluctuation strength. It turns out that, at sufficiently high pedestrian densities, lanes are destroyed by increasing the fluctuation strength (which is analogous to the temperature). However, instead of the expected transition from the "fluid" lane state to a disordered, "gaseous" state, a solid state is formed. It is characterized by a blocked situation with a regular (i.e. "crystallized" or "frozen") structure so that we call this paradoxial transition *"freezing by heating"* (see Fig. 7). Notably enough, the blocked state has a *higher* degree of order, although the internal energy is *increased* and the resulting state is *metastable* with respect to structural perturbations such as the exchange of oppositely moving particles [52]. Therefore, "freezing by heating" is just opposite to what one would expect for equilibrium systems, and different from fluctuation-driven ordering phenomena in some granular systems [140–142], where fluctuations lead from a disordered *metastable* to an ordered *stable* state [135].

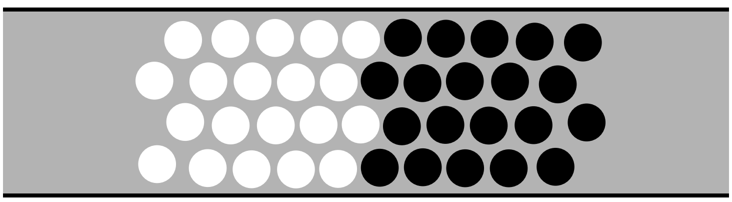

Figure 7: Noise-induced formation of a crystallized, "frozen" state in a periodic corridor used by oppositely moving pedestrians (after [52,95,99,136]).

The precondition for the unusual freezing-by-heating transition are the driving term $v_i^0\boldsymbol{e}_i^0/\tau_i$ and the dissipative friction $-\boldsymbol{v}_i/\tau_i$, while the sliding friction force is not required. Inhomogeneities in the channel diameter or other impurities which temporarily slow down pedestrians can further this transition at the respective places.

Finally note that a transition from fluid to blocked pedestrian counter flows is also observed, when a critical density is exceeded [52,59].

Transition to incoordination due to clogging. The simulated outflow from a room is well-coordinated and regular, if the desired velocities $v_i^0 = v_0$ are normal. However, for desired velocities above 1.5 m/s, i.e. for people in a rush, we find an irregular succession of arch-like blockings of the exit and avalanche-like bunches of leaving pedestrians, when the arches break (see Fig. 8a, b). This phenomenon is compatible with the empirical observations mentioned above and comparable to intermittent clogging found in granular flows through funnels or hoppers [115,116] (although this has been attributed to *static* friction between particles without remote interactions, and the transition to clogging has been observed for small enough openings rather than for a variation of the driving force).

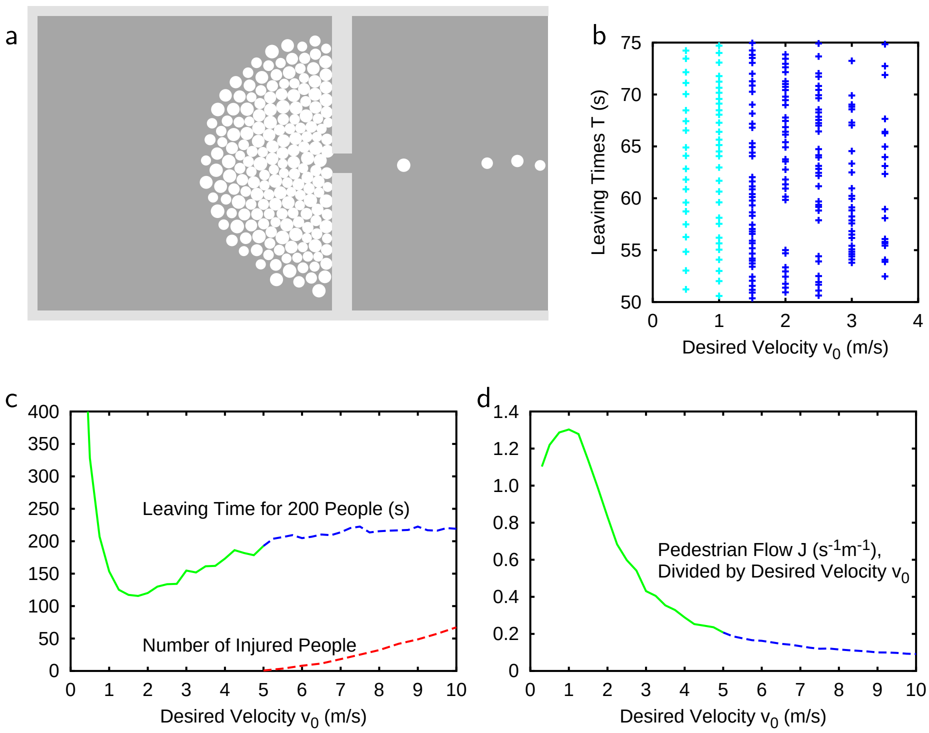

Figure 8: Simulation of pedestrians moving with identical desired velocity $v_i^0 = v_0$ towards the 1 m wide exit of a room of size 15 m × 15 m (from [53], see also [95,99,136]). a Snapshot of the scenario. Dynamic simulations are available at `http://angel.elte.hu/~panic/`. b Illustration of leaving times of pedestrians for various desired velocities v_0. Irregular outflow due to clogging is observed for high desired velocities ($v_0 \geq 1.5$ m/s, see dark plusses). c Under conditions of normal walking, the time for 200 pedestrians to leave the room decreases with growing v_0. Desired velocities higher than 1.5 m/s reduce the efficiency of leaving, which becomes particularly clear, when the outflow J is divided by the desired velocity (see d). This is due to pushing, which causes additional friction effects. Moreover, above a desired velocity of about $v_0 = 5$ m/s (– –), people are injured and become non-moving obstacles for others, if the sum of the magnitudes of the radial forces acting on them divided by their circumference exceeds a pressure of 1600 N/m [98].

"Faster-is-slower effect" due to impatience. Since clogging is connected with delays, trying to move faster (i.e., increasing v_i^0) can cause a smaller average speed of leaving, if the friction parameter κ is large enough (see Fig. 8c, d). This "faster-is-slower effect" is particularly tragic in the presence of fires, where fleeing people sometimes reduce their own chances of survival. The related fatalities can be estimated by the number of pedestrians reached by the fire front (see Fig. 9).

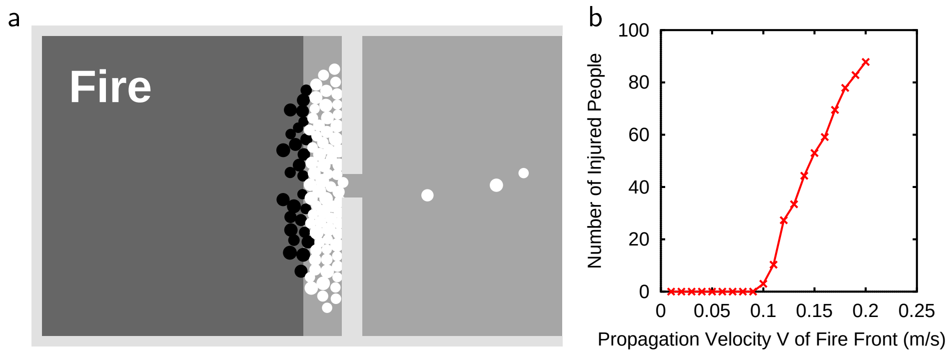

Figure 9: Simulation of $N = 200$ individuals fleeing from a linear fire front, which propagates from the left to the right wall with velocity V, starting at time $t = 5$ s [99] (for a Java simulation applet, see `http://angel.elte.hu/˜panic/`). a Snapshot of the scenario for a 15 m × 15 m large room with one door of width 1 m. The fire is indicated by dark grey color. Pedestrians reached by the fire front are injured and symbolized by black disks, while the white ones are still active. The socio-psychological effect of the fire front is assumed 10 times stronger than that of a normal wall ($A_F = 10A_i$). b Number of injured persons (casualities) as a function of the propagation velocity V of the fire front, averaged over 10 simulation runs. Up to a critical propagation velocity V_{crit} (here: about 0.1 m/s), nobody is injured. However, for higher velocities, we find a fast increase of the number of casualties with increasing V. The transition is continuous.

Since our friction term has, on average, no deceleration effect in the crowd, if the walls are sufficiently remote, the arching underlying the clogging effect requires a *combination* of several effects:

1. slowing down due to a bottleneck such as a door and
2. strong inter-personal friction, which becomes dominant when pedestrians get too close to each other. It is noteworthy that the faster-is-slower effect also occurs when the sliding friction force changes continuously with the distance rather than being "switched on" at a certain distance r_j as in the model above.

The danger of clogging can be minimized by avoiding bottlenecks in the construction of stadia and public buildings. Notice, however, that jamming can also occur at widenings of escape routes! This surprising result is illustrated in Fig. 10. It originates from disturbances due to pedestrians, who try to overtake each other and expand in the wide area because of their repulsive interactions. They squeeze into the main stream again at the end of the widening, which acts like a bottleneck and leads to jamming. The corresponding drop of efficiency E is more pronounced,

1. if the corridor is narrow,
2. if the pedestrians have different or high desired velocities, and
3. if the pedestrian density in the corridor is high.

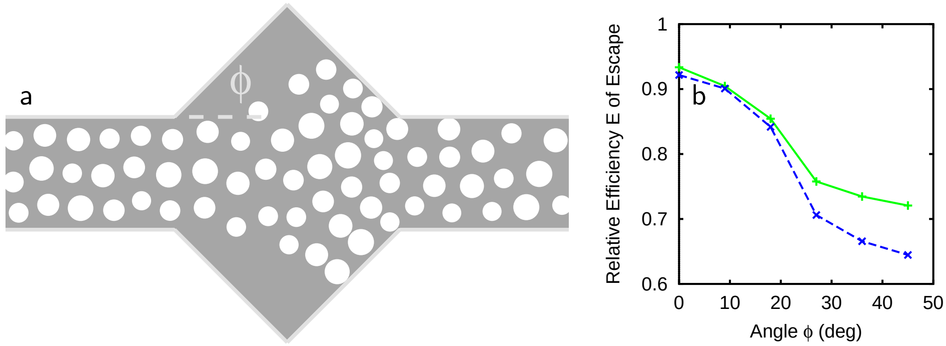

Figure 10: Simulation of an escape route with a wider area (from [53,99], see also the Java applets supplied at `http://angel.elte.hu/~panic/`). a Illustration of the scenario with $v_i^0 = v_0 = 2$ m/s. The corridor is 3 m wide and 15 m long, the length of the triangular pieces in the middle being $2{\times}3$ m$\,=6$ m. Pedestrians enter the simulation area on the left-hand side with an inflow of $J = 5.5$ s^{-1}m^{-1} and flee towards the right-hand side. b Efficiency of leaving as a function of the angle ϕ characterizing the width of the central zone, i.e., the difference from a linear corridor. The relative efficiency $E = \langle \boldsymbol{v}_i \cdot \boldsymbol{e}_i^0 \rangle / v_0$ measures the average velocity along the corridor compared to the desired velocity and lies between 0 and 1 (—). While it is almost one (i.e., maximal) for a linear corridor ($\phi = 0$), the efficiency drops by about 20%, if the corridor contains a widening. The decrease of efficiency E is even more pronounced in the area of the widening where pedestrian flow is most irregular (– –).

"Phantom panics". Sometimes, panics have occured *without* any comprehensible reasons such as a fire or another threatening event (e.g., in Moscow, 1982; Innsbruck, 1999). Due to the "faster-is-slower effect", panics can be triggered by small pedestrian counterflows [78], which cause delays to the crowd intending to leave. Consequently, stopped pedestrians in the back, who do not see the reason for the temporary slowdown, are getting impatient and pushy. In accordance with observations [43,18], one may describe this by increasing the desired velocity, for example, by the formula

$$v_i^0(t) = [1 - n_i(t)] v_i^0(0) + n_i(t) v_i^{\max} \,. \tag{11}$$

Herein, $v_i^{\max}$ is the maximum desired velocity and $v_i^0(0)$ the initial one, corresponding to the expected velocity of leaving. The time-dependent parameter

$$n_i(t) = 1 - \frac{\overline{v}_i(t)}{v_i^0(0)} \tag{12}$$

reflects the nervousness, where $\overline{v}_i(t)$ denotes the average speed into the desired direction of motion. Altogether, long waiting times increase the desired velocity, which can produce inefficient outflow. This further increases the waiting times, and so on, so that this tragic feedback can eventually trigger so high pressures that people are crushed or falling and trampled. It is, therefore, imperative, to have sufficiently wide exits and to prevent counterflows, when big crowds want to leave [53].

Ignorance of available exits. Finally, we investigate a situation in which pedestrians are trying to leave a smoky room, but first have to find one of the invisible exits (see Fig. 11a). Each pedestrian i may either select an individual direction $\boldsymbol{e}_i$ or follow the average direction $\langle \boldsymbol{e}_j^0(t)\rangle_i$ of his neighbours j in a certain radius R_i [143,144], or try a mixture of both. We assume that both options are weighted with the nervousness n_i:

$$\boldsymbol{e}_i^0(t) = \mathcal{N}\left[(1-n_i)\,\boldsymbol{e}_i + n_i\,\langle \boldsymbol{e}_j^0(t)\rangle_i\right] , \qquad (13)$$

where $\mathcal{N}(\boldsymbol{z}) = \boldsymbol{z}/\|\boldsymbol{z}\|$ denotes the normalization of a vector $\boldsymbol{z}$. As a consequence, we have individualistic behavior if n_i is low, but herding behavior if n_i is high. (In a somewhat different szenario, one may simulate a crowd with a proportion $(1-n)$ of individualists and a proportion n of herd followers.)

Our model suggests that neither individualistic nor herding behavior performs well (see Fig. 11b). Pure individualistic behavior means that each pedestrian finds an exit only accidentally, while pure herding behavior implies that the complete crowd is eventually moving into the same and probably blocked direction, so that available exits are not efficiently used, in agreement with observations. According to Figs. 11b and c, we expect optimal chances of survival for a certain mixture of individualistic and herding behavior, where individualism allows some people to detect the exits and herding guarantees that successful solutions are imitated by small groups of others. If pedestrians follow the walls instead of "reflecting" at them, we expect that herd following causes jamming and inefficient use of doors as well (see Fig. 8), while individualists moving in opposite directions obstruct each other.

5 Optimization of pedestrian flows

The emerging pedestrian flows decisively depend on the geometry of the boundaries. They can be simulated on a computer already in the planning phase of pedestrian facilities. Their configuration and shape can be systematically varied, e.g. by means of evolutionary algorithms [121,123,54] (see Fig. 12), and evaluated on the basis of particular mathematical performance measures [18,48]. Apart from the *efficiency* E with $0 \le E \le 1$ defined in formula (9), we can, for example, define the *measure of comfort* $C = (1-D)$ via the discomfort

$$D = \frac{1}{N}\sum_i \frac{\overline{(\boldsymbol{v}_i - \overline{\boldsymbol{v}_i})^2}}{\overline{(\boldsymbol{v}_i)^2}} = \frac{1}{N}\sum_i \left(1 - \frac{\overline{\boldsymbol{v}_i}^2}{\overline{(\boldsymbol{v}_i)^2}}\right) . \qquad (14)$$

The latter is again between 0 and 1 and reflects the frequency and degree of sudden velocity changes, i.e. the level of discontinuity of walking due to necessary avoidance maneuvers. Hence, the optimal configuration regarding the pedestrian requirements is the one with the highest values of efficiency and comfort.

During the optimization procedure, some or all of the following can be varied:

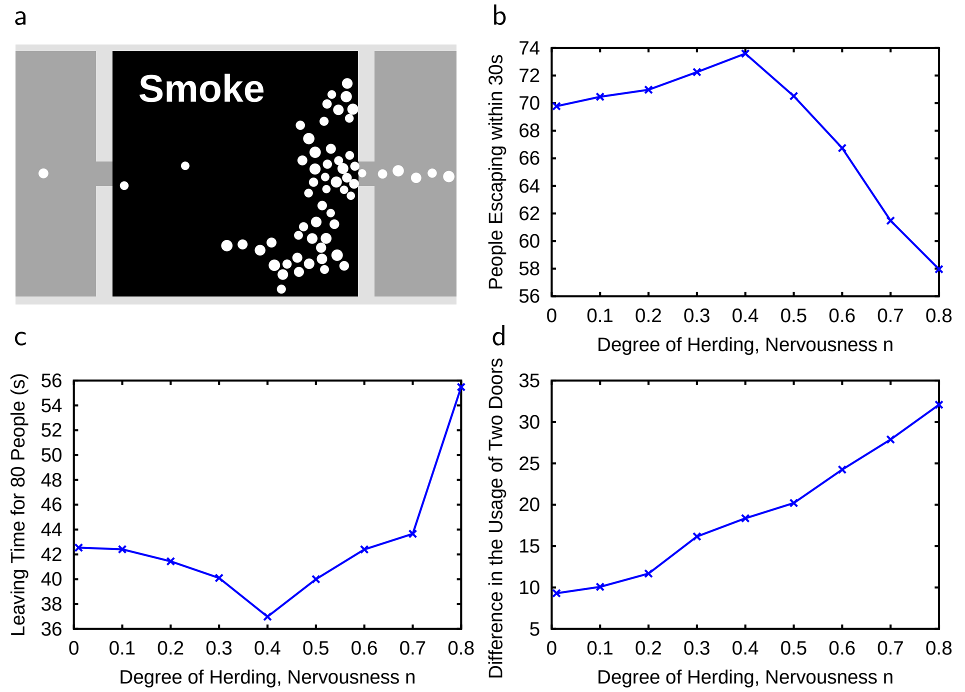

Figure 11: Simulation of $N = 90$ pedestrians trying to escape a smoky room of area $A = 15\text{m} \times 15$ m (black) through two smoke-hidden doors of 1.5 m width, which have to be found with a mixture of individualistic and herding behavior (from [53,95,99]). Java applets are available at `http://angel.elte.hu/~panic/`. a Snapshot of the simulation with $v_i^0 = v_0 = 5$ m/s. Initially, each pedestrian selects his or her desired walking direction randomly. Afterwards, a pedestrian's walking direction is influenced by the average direction of the neighbours within a radius of, for example, $R_i = R = 5$ m. The strength of this herding effect grows with increasing nervousness parameter $n_i = n$ and increasing value of $h = \pi R^2 \rho$, where $\rho = N/A$ denotes the pedestrian density. When reaching a boundary, the direction of a pedestrian is reflected. If one of the exits is closer than 2 m, the room is left. b Number of people who manage to escape within 30 s as a function of the nervousness parameter n. c Illustration of the time required by 80 individuals to leave the smoky room. If the exits are relatively narrow and the degree n of herding is small or large, leaving takes particularly long, so that only some of the people escape before being poisoned by smoke. Our results suggest that the best escape strategy is a certain compromise between following of others and an individualistic searching behavior. This fits well into experimental data on the efficiency of group problem solving [145–147], according to which groups normally perform better than individuals, but masses are inefficient in finding new solutions to complex problems. d Absolute difference $|N_1 - N_2|$ in the numbers N_1 and N_2 of persons leaving through the left exit or the right exit as a function of the degree n of herding. We find that pedestrians tend to jam up at one of the exits instead of equally using all available exits, if the nervousness is large.

1. the location and form of planned buildings,
2. the arrangement of walkways, entrances, exits, staircases, elevators, escalators, and corridors,
3. the shape of rooms, corridors, entrances, and exits,
4. the function and time schedule of room usage. (Recreation rooms or restaurants are continuously frequented, rooms for conferences or special events are mainly

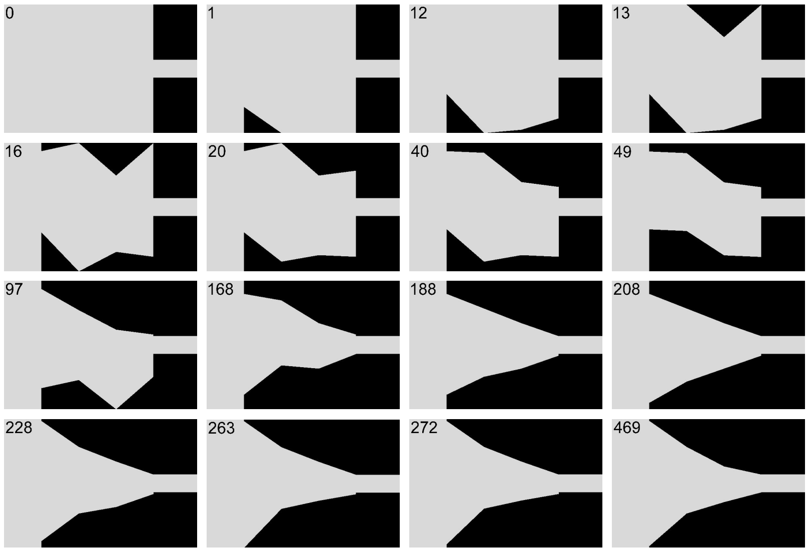

Figure 12: Different phases in the evolutionary optimization of a bottleneck (from [54,19]).

visited and left at peak periods, exhibition rooms or rooms for festivities require additional space for people standing around, and some areas are claimed by queues or through traffic.)

The proposed optimization procedure can not only be applied to the design of new pedestrian facilities but also to a reduction of existing bottlenecks by suitable modifications.

5.1 Normal situations

Here, we discuss four simple examples of how to improve some standard elements of pedestrian facilities ([18]; see Fig. 13):

1. At high pedestrian densities, the lanes of uniform walking direction tend to disturb each other: Impatient pedestrians try to use any gap for overtaking, which often leads to subsequent obstructions of the opposite walking directions. The lanes can be stabilized by series of trees or columns in the middle of the road (see Fig. 13a) which, in walking direction, looks similar to a wall (see Fig. 14). Also, it takes some detour to reach the other side of the permeable wall, which makes it less attractive to use gaps occuring in the opposite pedestrian stream.
2. The flow at bottlenecks can be improved by a funnel-shaped construction (see Fig. 13b) which, at the same time, allows one to save expensive space. Interestingly, the optimal form resulting from an evolutionary optimization is convex [54] (see Fig. 12).
3. A broader door does not necessarily lead to a proportional increase of pedestrian flow through it. It may rather lead to more frequent changes of the walking direction which are connected with temporary deadlock situations. Therefore, two doors close to the walls are more efficient than one single door with double width. By self-organization, each door is used by one walking direction [18,48–50], which is related to lane formation (see Fig. 15).

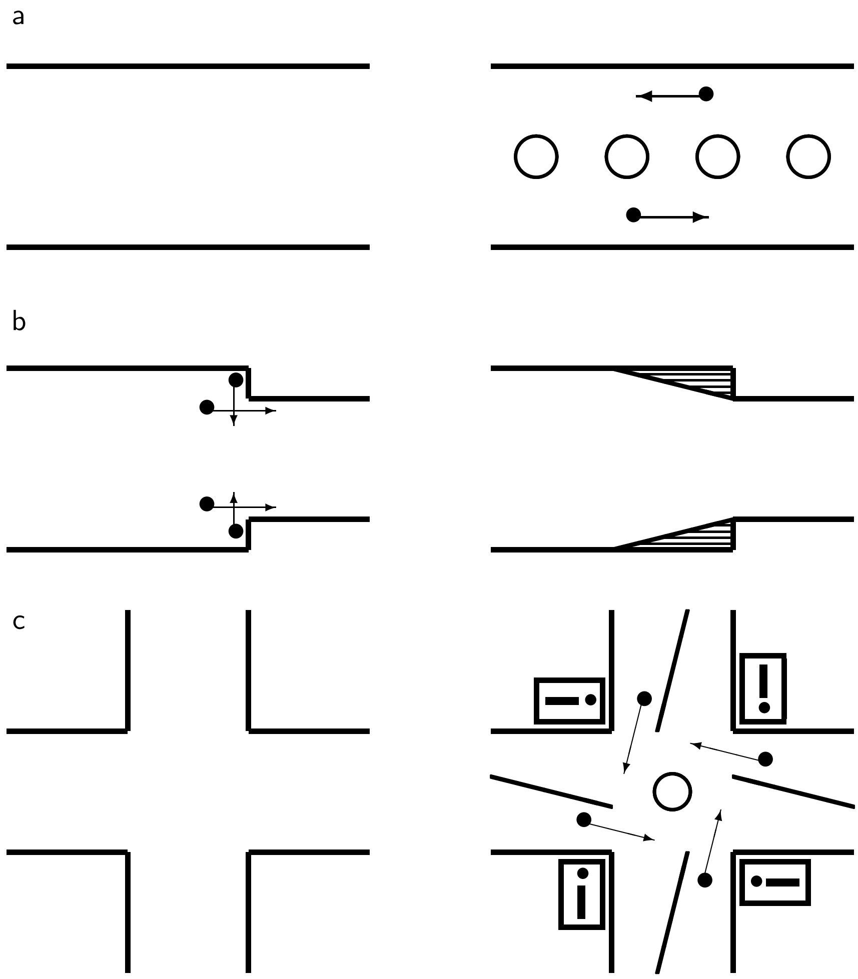

Figure 13: Conventional (left) and improved (right) elements of pedestrians facilities: a ways, b bottlenecks, and c intersections (from [18,19]). The exclamation marks stand for attraction effects (e.g. interesting posters above the street). Empty circles represent columns or trees, while full circles with arrows symbolize pedestrians and their walking directions.

4. Oscillatory changes of the walking direction and periods of standstill in between also occur when different flows *cross* each other. The loss of efficiency caused by this can be reduced by psychological guiding measures or railings initializing roundabout traffic (see Fig. 13c). Roundabout traffic can already be induced and stabilized by planting a tree in the middle of a crossing, because it surpresses the phases of “vertical” or “horizontal” motion in the intersection area. In our simulations this increased efficiency up to 13%.

The complex interaction between various flows can lead to completely unexpected results due to the nonlinearity of dynamics. (A very impressive and surprising result of evolutionary form optimization is presented by Klockgether and Schwefel in Ref. [120].) This means, planning of pedestrian facilities with conventional methods does not always guarantee the avoidance of big jams, serious obstructions, and catastrophic blockages (especially in emergency situations). In contrast, a skilful

Figure 14: These photographs of a pedestrian tunnel connecting two subways in Budapest at Deák tér illustrates that a series of columns acts similar to a wall and stabilizes lanes by preventing that their width exceeds half of the total width of the walkway.

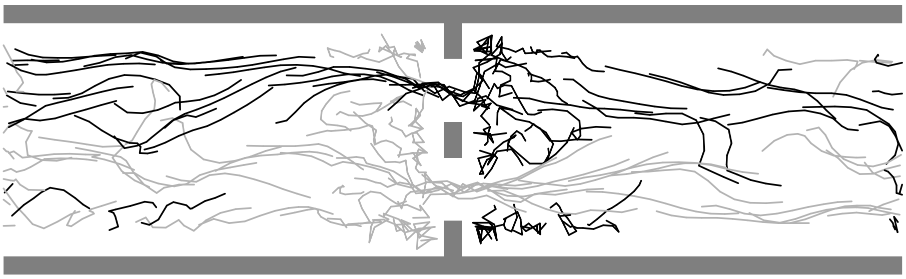

Figure 15: If two alternative passageways are available, pedestrians with opposite walking directions use different doors as a result of self-organization (after [18,19,45,48–50,95]).

flow optimization not only enhances efficiency but also saves space that can be used for kiosks, benches, or other purposes [18].

5.2 Panic situations

Similar design strategies can be developed for panic situations, where significantly improved outflows can be reached by columns placed asymmetrically in front of the exits [53,99]. These can prevent the build up of fatal pressures in exit areas and, thereby, also reduce injuries (see Fig. 16 and the Java applets at `http://angel.elte.hu/~panic/`). The asymmetrical placement helps to avoid equilibria of forces (blockages).

Additionally, one can guide people into the directions of usable exits by means of optical and acoustic stimuli, i.e. by suitable arrangements of light and sound sources.

6 Summary and outlook

We have developed a continuous pedestrian model based on plausible interactions, which is, due to its simplicity, robust with respect to parameter variations. It was pointed out that pedestrian dynamics shows various *collective phenomena, which*

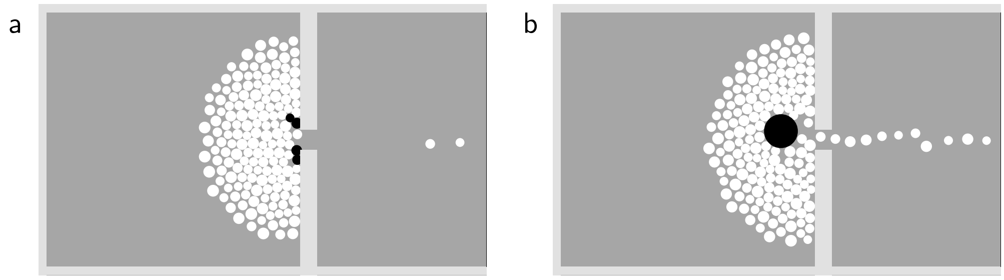

Figure 16: a In panicking crowds, high pressures build up due to physical interactions. This can injure people (black disks), who turn into obstacles for other pedestrians trying to leave (see also the lower curve in Fig. 8c). b A column in front of the exit (large black disk) can avoid injuries by taking up pressure from behind. It can also increase the outflow by 50%. In large exit areas used by many hundret people, several randomly placed columns are needed to subdivide the crowd and the pressure. An asymmetric configuration of the columns is most efficient, as it avoids equilibria of forces which may temporarily stop the outflow. (From [99].)

every simulation model should reproduce in order to be realistic. For example, in normal situations one finds

1. lane formation and
2. oscillatory flows through bottlenecks.

These and other empirical findings can be well described by our microscopic simulations of pedestrian streams based on a generalized force model. According to this model, the collective patterns of motion can be interpreted as self-organization phenomena, arising from the non-linear interactions among pedestrians.

We underline that self-organized flow patterns can significantly change the capacities of pedestrian facilities. They often lead to undesireable obstructions, but they can also be utilized to reach more efficient pedestrian flows with less space. Applications to the optimization of pedestrian facilities are, therefore, quite natural.

The proposed force model is also suitable for drawing conclusions about the possible mechanisms beyond escape panic (regarding an increase of the desired velocity, strong friction effects during physical interactions, and herding). After calibration of the model parameters to available data on pedestrian flows, we managed to reproduce many observed phenomena including

1. the breakdown of fluid lanes (“freezing by heating”),
2. the build up of fatal pressures,
3. clogging effects at bottlenecks,
4. jamming at widenings,
5. the “faster-is-slower effect”,
6. “phantom panics” triggered by counterflows and impatience, and
7. the ignorance of available exits due to herding.

The underlying behavior could be called “irrational”, as all of these effects decrease the chances of survival compared to normal pedestrian behavior. We were also able to simulate situations of dwindling resources and estimate the casualties (see Figs. 8c and 9). Therefore, the model could be used to test buildings for their suitability in emergency situations. It accounts for the considerably different dynamics both in normal and panic situations just by changing a single parameter $n_i = n$. In this way, we have proposed a consistent theoretical approach allowing a continuous switching between seemingly incompatible kinds of human behavior (individualistic, "rational" behavior vs. "irrational" panic behavior). Thereby, however, *we do not want to imply that individuals would always behave irrational in emergency situations.* It has been observed that, even in such situations individuals can behave highly self-controlled, coordinated, rational, and social. Our study just investigates the fundamental collective effects which fluctuations, increased desired velocities, and herding behavior can have, independently of whether all criteria of panics are fulfilled or not.

We believe that the above model can serve as an example linking collective behavior as a phenomenon of mass psychology (from the socio-psychological perspective) to the view of an emergent collective pattern of motion (from the perspective of physics). Our simulations suggest that the optimal behavior in escape situations is a suitable mixture of individualistic and herding behavior. This conclusion is probably transferable to many cases of problem solving in new and complex situations, where standard solutions fail. It may explain why both, individualistic and herding behaviors are common in human societies. For example, herding behavior is also relevant to fashion and stock market dynamics (see Refs. [99,148–153]). Apart from that, the competition of moving particles for limited space is analogous to the situation in various socio-economic systems, where individuals or other entities compete for limited resources as well. Therefore, conclusions from the above findings for self-driven many-particle systems reach far into the realm of the social, economic, and psychological sciences.

Finally, we are calling for quantitative data and, as far as possible, experimental studies of panic situations to make this model even more realistic. For example, one could include direction- and velocity-dependent interpersonal interactions, specify the individual variation of parameters, integrate acoustic information exchange, implement more complex strategies and interactions (also three-dimensional ones), or allow for switching of strategies. One should also complement the program by detailled fire and smoke propagation modules and model hazards, toxicity and behavioral reactions, as evacuation software tools like EXODUS do (see `http://fseg.gre.ac.uk/exodus`).

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References

1. B. D. Hankin and R. A. Wright, *Passenger flow in subways*, Operational Research Quarterly **9**, 81–88 (1958).
2. D. Oeding, *Verkehrsbelastung und Dimensionierung von Gehwegen und anderen Anlagen des Fußgängerverkehrs*, in *Straßenbau und Straßenverkehrstechnik*, Heft 22 (Bundesministerium für Verkehr, Abt. Straßenbau, Bonn, 1963).
3. L. A. Hoel, *Pedestrian travel rates in central business districts*, Traffic Engineering **38**, 10–13 (1968).
4. S. J. Older, *Movement of pedestrians on footways in shopping streets*, Traffic Engineering & Control **10**, 160–163 (1968).
5. P. D. Navin and R. J. Wheeler, *Pedestrian flow characteristics*, Traffic Engineering **39**, 31–36 (1969).
6. R. L. Carstens and S. L. Ring, *Pedestrian capacities of shelter entrances*, Traffic Engineering **41**, 38–43 (1970).
7. C. A. O'Flaherty and M. H. Parkinson, *Movement on a city centre footway*, Traffic Engineering & Control **13**, 434–438 (1972).
8. U. Weidmann, *Transporttechnik der Fußgänger*, in *Schriftenreihe des Instituts für Verkehrsplanung, Transporttechnik, Straßen- und Eisenbahnbau* (Institut für Verkehrsplanung, Transporttechnik, Straßen- und Eisenbahnbau, ETH Zürich, 1993).
9. M. R. Hill, *Walking, Crossing Streets, and Choosing Pedestrian Routes* (University of Nebraska, Lincoln, 1984).
10. M. Batty, *Predicting where we walk*, Nature **388**, 19–20 (1997).
11. J. J. Fruin, *Designing for pedestrians: A level-of-service concept*, in *Highway Research Record, Number 355: Pedestrians*, pp. 1–15 (Highway Research Board, Washington, D.C., 1971).
12. A. Polus , J. L. Schofer, and A. Ushpiz, *Pedestrian flow and level of service*, Journal of Transportation Engineering **109**, 46–56 (1983).
13. M. Mōri and H. Tsukaguchi, *A new method for evaluation of level of service in pedestrian facilities*, Transportation Research A **21**, 223–234 (1987).
14. H. Schubert, *Planungsmaßnahmen für den Fußgängerverkehr in den Städten*, in *Straßenbau und Straßenverkehrstechnik*, Heft 56 (Bundesministerium für Verkehr, Abt. Straßenbau, Bonn, 1967).
15. D. Boeminghaus, *Fußgängerbereiche + Gestaltungselemente* (Krämer, Stuttgart, 1982).
16. J. Pauls, *The movement of people in buildings and design solutions for means of egress*, Fire Technology **20**, 27–47 (1984).
17. W. H. Whyte, *City. Rediscovering the Center* (Doubleday, New York, 1988).
18. D. Helbing, *Verkehrsdynamik* (Springer, Berlin, 1997).
19. D. Helbing, P. Molnár, I. Farkas, and K. Bolay, *Self-organizing pedestrian movement*, Environment and Planning B **28**, xxx (2001).
20. H. Kirsch, *Leistungsfähigkeit und Dimensionierung von Fußgängerwegen*, in *Straßenbau und Straßenverkehrstechnik*, Heft 33 (Bundesministerium für Verkehr, Abt. Straßenbau, Bonn, 1964).
21. W. M. Predtetschenski and A. I. Milinski, *Personenströme in Gebäuden – Berechnungsmethoden für die Projektierung –* (Rudolf Müller, Köln-Braunsfeld, 1971).
22. Transportation Research Board, *Highway Capacity Manual*, Special Report 209 (Transportation Research Board, Washington, D.C., 1985).
23. D. G. Davis and J. P. Braaksma, *Adjusting for luggage-laden pedestrians in airport terminals*, Transportation Research A **22**, 375–388 (1988).

24. W. Brilon, M. Großmann, and H. Blanke, *Verfahren für die Berechnung der Leistungsfähigkeit und Qualität des Verkehrsablaufes auf Straßen*, in *Straßenbau und Straßenverkehrstechnik*, Heft 669 (Bundesministerium für Verkehr, Abt. Straßenbau, Bonn, 1993).
25. S. J. Yuhaski Jr., J. M. Macgregor Smith, *Modelling circulation systems in buildings using state dependent queueing models*, Queueing Systems **4**, 319–338 (1989).
26. J. R. Roy, *Queuing in spatially dispersed public facilities*, presented at the *IV. World Congress of the Regional Science Association International* (Palma de Mallorca, 1992).
27. G. G. Løvås, *Modelling and simulation of pedestrian traffic flow*, Transportation Research B **28**, 429–443 (1994).
28. D. Garbrecht, *Describing pedestrian and car trips by transition matrices*, Traffic Quarterly **27**, 89–109 (1973).
29. A. J. Mayne, *Some further results in the theory of pedestrians and road traffic*, Biometrika **41**, 375–389 (1954).
30. N. Ashford, M. O'Leary, and P. D. McGinity, *Stochastic modelling of passenger and baggage flows through an airport terminal*, Traffic Engineering & Control **17**, 207–210 (1976).
31. A. Borgers and H. Timmermans, *A model of pedestrian route choice and demand for retail facilities within inner-city shopping areas*, Geographical Analysis **18**, 115–128 (1986).
32. A. Borgers and H. Timmermans, *City centre entry points, store location patterns and pedestrian route choice behaviour: A microlevel simulation model*, Socio-Economic Planning Science **20**, 25–31 (1986).
33. H. Timmermans, X. van der Hagen, and A. Borgers, *Transportation systems, retail environments and pedestrian trip chaining behaviour: Modelling issues and applications*, Transportation Research B **26**, 45–59 (1992).
34. D. Helbing, *Stochastische Methoden, nichtlineare Dynamik und quantitative Modelle sozialer Prozesse*, Ph.D. thesis (University of Stuttgart, 1992, published by Shaker, Aachen, 1993).
35. L. F. Henderson, *The statistics of crowd fluids*, Nature **229**, 381–383 (1971).
36. L. F. Henderson, *On the fluid mechanics of human crowd motion*, Transportation Research **8**, 509–515 (1974).
37. L. F. Henderson and D. J. Lyons, *Sexual differences in human crowd motion*, Nature **240**, 353-355 (1972).
38. L. F. Henderson and D. M. Jenkins, *Response of pedestrians to traffic challenge*, Transportation Research **8**, 71–74 (1973).
39. R. L. Hughes, *The flow of large crowds of pedestrians*, Math. Comp. Simul. **53**, 367–370 (2000).
40. R. L. Hughes, *A continuum theory for the flow of pedestrians*, Transportation Research B, in print (2001).
41. D. Helbing, *A fluid-dynamic model for the movement of pedestrians*, Complex Systems **6**, 391–415 (1992).
42. S. P. Hoogendoorn and P. H. L. Bovy, *Gas-kinetic modelling and simulation of pedestrian flows*, Transportation Research Records **1710**, 28–36 (2000).
43. D. Helbing, *A mathematical model for the behavior of pedestrians*, Behavioral Science **36**, 298–310 (1991).
44. D. Helbing, *Traffic modeling by means of physical concepts*, in *Traffic and Granular Flow*, pp. 87–104, D. E. Wolf, M. Schreckenberg, and A. Bachem (Eds.) (World Scientific, Singapore, 1996).
45. D. Helbing, *Pedestrian dynamics and trail formation*, pp. 21–36, in *Traffic and Granular Flow '97*, M. Schreckenberg and D. E. Wolf (Eds.) (Springer, Singapore, 1998).

46. D. Helbing, P. Molnár, and F. Schweitzer, *Computer simulations of pedestrian dynamics and trail formation*, in *Evolution of Natural Structures*, pp. 229–234 (Sonderforschungsbereich 230, Stuttgart, 1994).
47. D. Helbing and P. Molnár, *Social force model for pedestrian dynamics*, Physical Review E **51**, 4282–4286 (1995).
48. D. Helbing and P. Molnár, *Self-organization phenomena in pedestrian crowds*, in *Self-Organization of Complex Structures: From Individual to Collective Dynamics*, pp. 569–577, F. Schweitzer (Ed.) (Gordon and Breach, London, 1997).
49. P. Molnár, *Modellierung und Simulation der Dynamik von Fußgängerströmen*, (Shaker, Aachen, 1996).
50. P. Molnár, *Microsimulation of pedestrian dynamics* in *Social Science Microsimulation*, J. Doran, N. Gilbert, U. Mueller, and K. Troitzsch (Eds.) (Springer, Berlin, 1996).
51. D. Helbing and T. Vicsek, *Optimal self-organization*, New Journal of Physics **1**, 13.1–13.17 (1999).
52. D. Helbing, I. Farkas, and T. Vicsek, *Freezing by heating in a driven mesoscopic system*, Physical Review Letters **84**, 1240–1243 (2000).
53. D. Helbing, I. Farkas, and T. Vicsek, *Simulating dynamical features of escape panic*, Nature **407**, 487–490 (2000).
54. K. Bolay, *Nichtlineare Phänomene in einem fluid-dynamischen Verkehrsmodell* (Master's thesis, University of Stuttgart, 1998).
55. V. J. Blue and J. L. Adler, *Emergent fundamental pedestrian flows from cellular automata microsimulation*, Transportation Research Records **1644**, 29–36 (1998).
56. V. J. Blue and J. L. Adler, *Using cellular automata microsimulation to model pedestrian movements*, in *Proceedings of the 14th International Symposium on Transportation and Traffic Theory*, pp. 235–254, A. Ceder (Ed.) (Pergamon, New York, N. Y., 1999).
57. M. Fukui and Y. Ishibashi, *Self-organized phase transitions in cellular automaton models for pedestrians*, Journal of the Physical Society of Japan **68**, 2861–2863 (1999).
58. M. Fukui and Y. Ishibashi, *Jamming transition in cellular automaton models for pedestrian on passageway*, Journal of the Physical Society of Japan **68**, 3738–3739 (1999).
59. M. Muramatsu, T. Irie, and T. Nagatani, *Jamming transition in pedestrian counter flow*, Physica A **267**, 487–498 (1999).
60. M. Muramatsu and T. Nagatani, *Jamming transition in two-dimensional pedestrian traffic*, Physica A **275**, 281–291 (2000).
61. M. Muramatsu and T. Nagatani, *Jamming transition of pedestrian traffic at a crossing with open boundaries*, Physica A **286**, 377–390 (2000).
62. H. Klüpfel, M. Meyer-König, J. Wahle, and M. Schreckenberg, *Microscopic simulation of evacuation processes on passenger ships*, in *Theory and Practical Issues on Cellular Automata*, S. Bandini and T. Worsch (Eds.) (Springer, London, 2000).
63. C. Burstedde, K. Klauck, A. Schadschneider, and J. Zittartz, *Simulation of pedestrian dynamics using a 2-dimensional cellular automaton*, Physica A, in print (2001).
64. S. Gopal and T. R. Smith, *NAVIGATOR: An AI-based model of human way-finding in an urban environment*, in *Spatial Choices and Processes*, pp. 169–200, M. M. Fischer, P. Nijkamp, and Y. Y. Papageorgiou (Eds.) (North-Holland, Amsterdam, 1990).
65. C. W. Reynolds, *Evolution of corridor following behavior in a noisy world*, in *From Animals to Animats 3: Proceedings of the Third International Conference on Simulation of Adaptive Behavior*, pp. 402–410, D. Cliff, P. Husbands, J.-A. Meyer, and S. Wilson (Eds.) (MIT Press, Cambridge, Massachusetts, 1994).

66. C. W. Reynolds, *Steering behaviors for autonomous characters*, in *Proceedings of the 1999 Game Developer's Conference*, pp. 763–782, A. Yu (Ed.) (Miller Freeman, San Fransisco, 1999).
67. T. Schelhorn, D. O'Sullivan, M. Haklay, and M. Thurstain-Goodwin, *STREETS: An agent-based pedestrian model*, presented at the conference *Computers in Urban Planning and Urban Management* (Venice, 1999).
68. K. H. Drager, G. Løvås, J. Wiklund, H. Soma, D. Duong, A. Violas, and V. Lanèrès, *EVACSIM—A comprehensive evacuation simulation tool*, in the *Proceedings of the 1992 Emergency Management and Engineering Conference*, pp. 101–108 (Society for Computer Simulation, Orlando, Florida, 1992).
69. M. Ebihara, A. Ohtsuki, and H. Iwaki, *A model for simulating human behavior during emergency evacuation based on classificatory reasoning and certainty value handling*, Microcomputers in Civil Engineering **7**, 63–71 (1992).
70. N. Ketchell, S. Cole, D. M. Webber, C. A. Marriott, P. J. Stephens, I. R. Brearley, J. Fraser, J. Doheny, and J. Smart, *The EGRESS Code for human movement and behaviour in emergency evacuations*, in *Engineering for Crowd Safety*, pp. 361–370, R. A. Smith and J. F. Dickie (Eds.) (Elsevier, Amsterdam, 1993).
71. S. Okazaki and S. Matsushita, *A study of simulation model for pedestrian movement with evacuation and queuing*, in *Engineering for Crowd Safety*, pp. 271–280, R. A. Smith and J. F. Dickie (Eds.) (Elsevier, Amsterdam, 1993).
72. G. K. Still, *New computer system can predict human behaviour response to building fires*, Fire **84**, 40–41 (1993).
73. G. K. Still, *Crowd Dynamics* (Ph.D. thesis, University of Warwick, 2000).
74. P. A. Thompson and E. W. Marchant, *Modelling techniques for evacuation*, in *Engineering for Crowd Safety*, pp. 259–269, R. A. Smith and J. F. Dickie (Eds.) (Elsevier, Amsterdam, 1993).
75. G. G. Løvås, *On the importance of building evacuation system components*, IEEE Transactions on Engineering Management **45**, 181–191 (1998).
76. H. W. Hamacher and S. A. Tjandra, *Modelling Emergency Evacuation—An Overview*, to be published (2001).
77. J. P. Keating, *The myth of panic*, Fire Journal, 57–61+147 (May/1982).
78. D. Elliott and D. Smith, *Football stadia disasters in the United Kingdom: Learning from tragedy?*, Industrial & Environmental Crisis Quarterly **7**(3), 205–229 (1993).
79. B. D. Jacobs and P. 't Hart, *Disaster at Hillsborough Stadium: a comparative analysis*, in *Hazard Management and Emergency Planning*, Chap. 10, D. J. Parker and J. W. Handmer (Eds.) (James & James Science, London, 1992).
80. D. Canter (Ed.), *Fires and Human Behaviour*, (David Fulton, London, 1990).
81. A. Mintz, *Non-adaptive group behavior*, The Journal of Abnormal and Normal Social Psychology **46**, 150–159 (1951).
82. D. L. Miller, *Introduction to Collective Behavior*, Fig. 3.3 and Chap. 9 (Wadsworth, Belmont, CA, 1985).
83. J. S. Coleman, *Foundations of Social Theory*, Chaps. 9 and 33 (Belkamp, Cambridge, MA, 1990).
84. N. R. Johnson, *Panic at "The Who Concert Stampede": An empirical assessment*, Social Problems **34**(4), 362–373 (1987).
85. G. LeBon, *The Crowd* (Viking, New York, 1960 [1895]).
86. E. Quarantelli, *The behavior of panic participants*, Sociology and Social Research **41**, 187–194 (1957).
87. N. J. Smelser, *Theory of Collective Behavior*, (The Free Press, New York, 1963).
88. R. Brown, *Social Psychology* (The Free Press, New York, 1965).
89. R. H. Turner and L. M. Killian, *Collective Behavior* (Prentice Hall, Englewood Cliffs, NJ, 3rd ed., 1987).

90. J. L. Bryan, *Convergence clusters*, Fire Journal, 27–30+86–90 (Nov./1985).
91. R. Axelrod and W. D. Hamilton, *The evolution of cooperation*, Science **211**, 1390–1396 (1981).
92. R. Axelrod and D. Dion, *The further evolution of cooperation*, Science **242**, 1385–1390 (1988).
93. N. S. Glance and B. A. Huberman, *The dynamics of social dilemmas*, Scientific American **270**, 76–81 (1994).
94. T. Arns, Video films of pedestrian crowds, (Wannenstr. 22, 70199 Stuttgart, 1993).
95. D. Helbing, *Traffic and related self-driven many-particle systems*, e-print cond-mat/0012229, to appear in Reviews of Modern Physics (2001).
96. J. Ganem, *A behavioral demonstration of Fermat's principle*, The Physics Teacher **36**, 76–78 (1998).
97. D. Helbing, J. Keltsch, and P. Molnár, *Modelling the evolution of human trail systems*, Nature **388**, 47–50 (1997).
98. R. A. Smith and J. F. Dickie (Eds.), *Engineering for Crowd Safety* (Elsevier, Amsterdam, 1993).
99. D. Helbing, I. J. Farkas, and T. Vicsek, *Crowd disasters and simulation of panic situations*, in *Science of Disaster: Climate Disruptions, Heart Attacks and Market Crashes*, A. Bunde, J. Kropp, and H. J. Schellnhuber (Eds.) (Springer, Berlin, 2001).
100. J. S. Coleman, *Introduction to Mathematical Sociology*, pp. 361–375 (The Free Press of Glencoe, New York, 1964).
101. J. S. Coleman and J. James, *The equilibrium size distribution of freely-forming groups*, Sociometry **24**, 36–45 (1961).
102. L. A. Goodman, *Mathematical methods for the study of systems of groups*, American Journal of Sociology **70**, 170–192 (1964).
103. H. H. Kelley, J. C. Condry Jr., A. E. Dahlke, and A. H. Hill, *Collective behavior in a simulated panic situation*, Journal of Experimental Social Psychology **1**, 20–54 (1965).
104. L. P. Kadanoff, *Simulating hydrodynamics: A pedestrian model*, Journal of Statistical Physics **39**, 267–283 (1985).
105. H. E. Stanley and N. Ostrowsky (Eds.), *On Growth and Form* (Martinus Nijhoff, Boston, 1986).
106. H.-H. Stølum, *River meandering as a self-organization process*, Nature **271**, 1710–1713 (1996).
107. I. Rodríguez-Iturbe and A. Rinaldo, *Fractal River Basins: Chance and Self-Organization* (Cambridge University, Cambridge, England, 1997).
108. G. Caldarelli, *Cellular models for river networks*, e-print cond-mat/0011086 (2000).
109. S. B. Santra, S. Schwarzer, and H. Herrmann, *Fluid-induced particle segregation in sheared granular assemblies*, Physical Review E **54**, 5066–5072 (1996).
110. H. A. Makse, S. Havlin, P. R. King, and H. E. Stanley, *Spontaneous stratification in granular mixtures*, Nature **386**, 379–382 (1997).
111. K. Yoshikawa, N. Oyama, M. Shoji, and S. Nakata, *Use of a saline oscillator as a simple nonlinear dynamical system: Rhythms, bifurcation, and entrainment*, American Journal of Physics **59**, 137–141 (1991).
112. X.-L. Wu, K. J. Måløy, A. Hansen, M. Ammi, and D. Bideau, *Why hour glasses tick*, Physical Review Letters **71**, 1363–1366 (1993).
113. T. L. Pennec, K. J.Måløy, A. Hansen, M. Ammi, D. Bideau D, and X.-L. Wu, *Ticking hour glasses: Experimental analysis of intermittent flow*, Physical Review E **53**, 2257–2264 (1996).
114. M. R. Virkler and S. Elayadath, *Pedestrian density characteristics and shockwaves*, in *Proceedings of the Second International Symposium on Highway Capacity*, Vol. 2, pp. 671–684, R. Akçelik (Ed.) (Transportation Research Board, Washington, D.C., 1994).

115. G. H. Ristow and H. J. Herrmann, *Density patterns in two-dimensional hoppers*, Physical Review E **50**, R5–R8 (1994).
116. D. E. Wolf and P. Grassberger (Eds.), *Friction, Arching, Contact Dynamics* (World Scientific, Singapore, 1997).
117. D. Helbing, *Boltzmann-like and Boltzmann-Fokker-Planck equations as a foundation of behavioral models*, Physica A **196**, 546–573 (1993).
118. D. Helbing, *A mathematical model for the behavior of individuals in a social field*, Journal of Mathematical Sociology **19**, 189–219 (1994).
119. D. Helbing, *Quantitative Sociodynamics. Stochastic Methods and Models of Social Interaction Processes* (Kluwer Academic, Dordrecht, 1995).
120. J. Klockgether and H.-P. Schwefel, *Two-phase nozzle and hollow core jet experiments*, in *Proceedings of the Eleventh Symposium on Engineering Aspects of Magnetohydrodynamics*, pp. 141–148, D. G. Elliott (Ed.) (California Institute of Technology, Pasadena, CA, 1970).
121. I. Rechenberg, *Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution* (Frommann-Holzboog, Stuttgart, 1973).
122. H.-P. Schwefel, *Numerische Optimierung von Computer-Modellen mittels Evolutionsstrategien* (Birkhäuser, Stuttgart, 1977).
123. T. Baeck, *Evolutionary Algorithms in Theory and Practice* (Oxford University Press, New York, 1996).
124. D. Helbing, *Physikalische Modellierung des dynamischen Verhaltens von Fußgängern* (Master's thesis, Georg-August University Göttingen, 1990).
125. D. Helbing, *A mathematical model for behavioral changes by pair interactions*, in *Economic Evolution and Demographic Change. Formal Models in Social Sciences*, pp. 330–348, G. Haag, U. Mueller, and K. G. Troitzsch (Eds.) (Springer, Berlin, 1992).
126. D. Helbing, *Stochastic and Boltzmann-like models for behavioral changes, and their relation to game theory*, Physica A **193**, 241–258 (1993).
127. D. Helbing, *A stochastic behavioral model and a 'microscopic' foundation of evolutionary game theory*, Theory and Decision **40**, 149–179 (1996).
128. N. E. Miller, *Experimental studies of conflict*, in *Personality and the behavior disorders*, Vol. 1, J. McV. Hunt (Ed.) (Ronald, New York, 1944).
129. N. E. Miller, *Liberalization of basic S-R-concepts: Extension to conflict behavior, motivation, and social learning* in *Psychology: A Study of Science*, Vol. 2, S. Koch (Ed.) (McGraw Hill, New York, 1959).
130. W. H. Herkner, *Ein erweitertes Modell des Appetenz-Aversions-Konflikts*, Z. Klin. Psychol. **4**, 50–60 (1975).
131. K. Lewin, *Field Theory in Social Science*, (Harper & Brothers, New York, 1951).
132. D. Helbing and B. Tilch, *Generalized force model of traffic dynamics*, Physical Review E **58**, 133–138 (1998).
133. M. Treiber, A. Hennecke, and D. Helbing, *Congested traffic states in empirical observations and microscopic simulations*, Physical Review E **62**, 1805–1824 (2000).
134. A. K. Dewdney, *Diverse personalities search for social equilibrium at a computer party*, Scientific American **257**(9), 104–107 (1987).
135. D. Helbing and T. Platkowski, *Self-organization in space and induced by fluctuations*, International Journal of Chaos Theory and Applications **5**, 25–39 (2000).
136. D. Helbing, *Die wundervolle Welt aktiver Vielteilchensysteme*, Physikalische Blätter **57**, 27–33 (2001).
137. F. Schweitzer, K. Lao, and F. Family, *Active random walkers simulate trunk trail formation by ants*, BioSystems **41**, 153–166 (1997).
138. D. Helbing, F. Schweitzer, J. Keltsch, and P. Molnár, *Active walker model for the formation of human and animal trail systems*, Physical Review E **56**, 2527–2539 (1997).

139. Y. L. Duparcmeur, H. Herrmann, and J. P. Troadec, *Spontaneous formation of vortex in a system of self motorised particles*, Journal de Physique I France **5**, 1119–1128 (1995).
140. J. Gallas, H. J. Herrmann, and S. Sokołowski, *Convection cells in vibrating granular media*, Physical Review Letters **69**, 1371–1374 (1992).
141. P. B. Umbanhowar, F. Melo, and H. L. Swinney, *Localized excitations in a vertically vibrated granular layer*, Nature **382**, 793–796 (1996).
142. A. Rosato, K. J. Strandburg, F. Prinz, and R. H. Swendsen, *Why the Brazil nuts are on top: Size segregation of particulate matter by shaking*, Physical Review Letters **58**, 1038–1041 (1987).
143. T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, *Novel type of phase transition in a system of self-driven particles*, Physical Review Letters **75**, 1226–1229 (1995).
144. A. Czirók, M. Vicsek, and T. Vicsek, *Collective motion of organisms in three dimensions*, Physica A **264**, 299–304 (1999).
145. N. H. Anderson, *Group performance in an anagram task*, Journal of Social Psychology **55**, 67–75 (1961).
146. H. H. Kelley and J. W. Thibaut, *Group problem solving*, in *The Handbook of Social Psychology*, Vol. 4, G. Lindzey and E. Aronson (Eds.) (Addison-Wesley, Reading, MA, 1969).
147. P. R. Laughlin, N. L. Kerr, J. H. Davis, H. M. Halff, and K. A. Marciniak, *Group size, member ability, and social decision schemes on an intellective task*, Journal of Personality and Social Psychology **31**, 522–535 (1975).
148. L. Mann, T. Nagel, and P. Dowling, *A study of economic panic: The "run" on the Hindmarsh building society*, Sociometry **39**(3), 223–235 (1976).
149. M. Youssefmir, B. A. Huberman, and T. Hogg, *Bubbles and market crashes*, Comp. Econ. **12**, 97–114 (1998).
150. J. D. Farmer, *Market force, ecology, and evolution*, e-print adap-org/9812005, submitted to the Journal of Economic Behavior and Organization (1998).
151. T. Lux and M. Marchesi, *Scaling and criticality in a stochastic multi-agent model of a financial market*, Nature **397**, 498–500 (1999).
152. J.-P. Bouchaud, A. Matacz, and M. Potters, *The leverage effect in financial markets: Retarded volatility and market panic*, e-print cond-mat/0101120 (2001).
153. D. Sornette and J. V. Andersen, *Quantifying Herding During Speculative Financial Bubbles*, e-print cond-mat/0104341 (2001).